

Production of Neutral and Doubly Charged Partners of $D_{s0}^+(2317)$ Revisited

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Rates for productions of neutral and doubly charged partners of $D_{s0}^+(2317)$ in B meson decays are studied by using a hard D meson approximation in the infinite momentum frame, and the results are of the same order of magnitude as that of $D_{s0}^+(2317)$. Because the bottom-strange $X^\pm(5568)$ which can be interpreted as iso-triplet bottom partners of $D_{s0}^+(2317)$ have recently been discovered, observations of neutral and doubly charged partners of $D_{s0}^+(2317)$ are strongly desired.

The charm-strange scalar meson $D_{s0}^+(2317)$ was discovered in the $D_s^+\pi^0$ mass distribution in inclusive e^+e^- annihilations [1, 2] and also in exclusive B decays [3]. However, no signal of $D_{s0}^+(2317)$ was observed in the $D_s^{*+}\gamma$ channel, (its indication was once reported in [3] but nothing after that). Thus, the measured value of the ratio of rates

$$R_{D_{s0}^+(2317)} = \frac{\Gamma(D_{s0}^+(2317) \rightarrow D_s^{*+}\gamma)}{\Gamma(D_{s0}^+(2317) \rightarrow D_s^+\pi^0)} \quad (1)$$

is now given by [4]

$$R_{D_{s0}^+(2317)}^{\text{exp}} < 0.059. \quad (2)$$

The above decay property of $D_{s0}^+(2317)$ favors its assignment to an iso-triplet state [5, 6], because there exists a hierarchy of hadron interactions, $|isospin\ conserving\ ones| \sim O(1) \gg |electromagnetic\ ones| \sim O(\sqrt{\alpha}) \gg |isospin\ non-conserving\ ones| \sim O(\alpha)$ [7], where α is the fine structure constant. In addition, we have seen numerically that decay properties of the charmed vector mesons, $D^{*+,0}$ and D_s^{*+} , are compatible with the above hierarchy [8], in particular, the isospin non-conserving $D_s^{*+} \rightarrow D_s^+\pi^0$ decay is much weaker than the radiative $D_s^{*+} \rightarrow D_s^+\gamma$ as expected by the above hierarchy and our theoretical ratio of rates $\Gamma(D_s^{*+} \rightarrow D_s^+\pi^0)/\Gamma(D_s^{*+} \rightarrow D_s^+\gamma)$, which is dependent on the $\eta\eta'$ mixing angle θ_P , is consistent with the measured one [4], $[\Gamma(D_s^{*+} \rightarrow D_s^+\pi^0)/\Gamma(D_s^{*+} \rightarrow D_s^+\gamma)]_{\text{exp}} = 0.062 \pm 0.008$, when $\theta_P = -11.4^\circ$ [4] is taken. It means that the hierarchy is really working. Under this condition, the assignment of $D_{s0}^+(2317)$ to an iso-triplet state implies that the $D_{s0}^+(2317) \rightarrow D_s^+\pi^0$ decay is isospin conserving and therefore, Eq. (2) is compatible with the early half of the above hierarchy, while its assignment to an iso-singlet state [9, 10] implies that the decay is isospin non-conserving, so that Eq. (2) is against the later half of the hierarchy. In addition, very recently, iso-triplet bottom-strange $X^\pm(5568)$ have been discovered in the $B_s^0\pi^\pm$ channels (based on 10.4 fb^{-1} $p\bar{p}$ collision data) by the D0 collaboration [11], (while the LHCb [12] have not observed them in 3 fb^{-1} pp collision data). If such states truly exist, they can be interpreted as the bottom partners of $D_{s0}^+(2317)$ [13], and it implies that the assignment of $D_{s0}^+(2317)$ to an iso-triplet state is quite natural.

When $D_{s0}^+(2317)$ is truly an iso-triplet meson, it cannot be realized by any ordinary $\{c\bar{s}\}$ state. Although one might expect that it could be realized by a $\{DK\}$ molecular state, such an expectation would not be realistic, as long as the OZI rule and the isospin $SU_I(2)$ symmetry work, as seen below. The $\{D^+K^+\}$ and $\{D^0K^0\}$ systems cannot have any $\{q\bar{q}\}$ meson exchange in the t -channel under the OZI rule [14], because all the species of constituent quarks in each of these systems are different from each other, and therefore, no connected $\{q\bar{q}\}$ exchange diagram exists. It implies that there exists no ordinary meson exchange as the origin of the binding force between D and K , as long as the systems are iso-triplet states [15], and hence, it is hard to consider that $D_{s0}^+(2317)$ is an iso-triplet $\{DK\}$ molecular state. In this way, we consider that $D_{s0}^+(2317)$ is the $I_3 = 0$ component (\hat{F}_I^+) of iso-triplet charm-strange scalar *tetra-quark* mesons, ($\hat{F}_I^0, \hat{F}_I^+, \hat{F}_I^{++}$). In this case, however, one might ask how to reconcile a theoretical rate for its isospin conserving decay with the measured narrow width of $D_{s0}^+(2317)$ [4]. Nevertheless, such a problem is not necessarily serious, because tetra-quark states have a variety of color and spin configurations, and therefore, the wave function overlap between \hat{F}_I^+ and $D_s^+\pi^0$ and hence the $\hat{F}_I^+D_s^-\pi^0$ coupling strength can be suppressed, when the ideally mixed $[qq][\bar{q}\bar{q}]$, ($q = u, d, s, c$) are taken as the *scalar* tetra-quark mesons [6]. A problem is that the Belle collaboration [16] have not observed any indication of \hat{F}_I^0 and \hat{F}_I^{++} (z^0 and z^{++} , respectively, in [16]), in spite of our theoretical expectation of their existence and observation in B_u^+ and B_d^0 decays [17] in addition to the observation of $X^\pm(5568)$ as the candidates of iso-triplet bottom partners of $D_{s0}^+(2317)$. Therefore, we here re-consider explicitly their production rates in B decays.

To this aim, we review shortly our previous work on productions [17] of $D_{s0}^+(2317)$ and its partners. Productions of $\hat{F}_I^+ = D_{s0}^+(2317)$ and its iso-singlet partner (\hat{F}_0^+) in inclusive e^+e^- annihilations within the framework of minimal $\{q\bar{q}\}$ pair creation is depicted by Fig. 1(a), as in [18]. In this case, production rate of the iso-triplet \hat{F}_I^+ is much higher than that of \hat{F}_0^+ , because an iso-triplet $\{n\bar{n}\}$ pair couples more strongly to a photon than an iso-singlet one, where

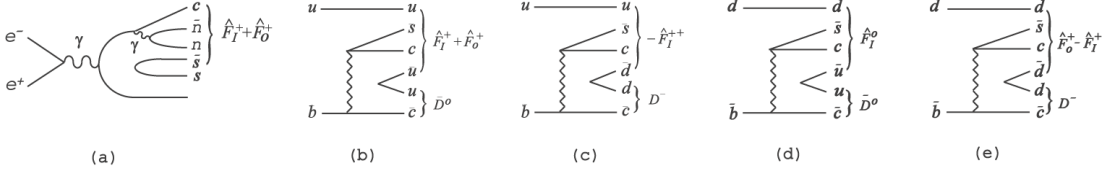


Fig. 1. Productions of charm-strange scalar tetra-quark mesons. (a) Productions of \hat{F}_I^{++} and \hat{F}_0^{++} through electromagnetic interactions in inclusive e^+e^- annihilations, (b) and (e) productions of \hat{F}_I^{++} and \hat{F}_0^{++} , and (c) and (d) \hat{F}_I^{++} and \hat{F}_I^0 , respectively, in B decays, where $n = u, d$.

$n = u, d$, and hence, it is easier to observe \hat{F}_I^{++} than \hat{F}_0^{++} in inclusive e^+e^- annihilations. It is consistent with the fact that the Babar and CLEO collaborations have observed $\hat{F}_I^{++} = D_{s0}^+(2317)$ in the $D_s^+\pi^0$ channel but not any excess (as the indication of its iso-singlet partner) around the mass of $D_{s0}^+(2317)$ in the $D_s^{*+}\gamma$ spectrum [1, 2]. In addition, we cannot find any diagram to depict productions of doubly charged and neutral partners (\hat{F}_I^{++} and \hat{F}_I^0) of $D_{s0}^+(2317)$ in inclusive e^+e^- annihilations within the same framework, so that we do not expect their observation in inclusive e^+e^- annihilations. It seems to be consistent with the fact that the Babar collaboration did not observe any indication of doubly charged and neutral partners of $D_{s0}^+(2317)$ in inclusive e^+e^- annihilations [19].

Productions of $\hat{F}_I^{++} = D_{s0}^+(2317)$ and \hat{F}_0^{++} in B decays are depicted by the quark-line diagrams, Figs. 1(b) and (e), and its doubly charged and neutral partners in B decays by the diagrams, Figs. 1(c) and (d), respectively. As seen in these diagrams, amplitudes for these decays are non-factorizable, in contrast to those for the $B \rightarrow D_s^+$ (or D_s^{*+}) \bar{D} decays whose amplitudes are factorizable and can be approximately calculated in accordance with the vacuum insertion prescription. By the way, it is well-known that rates for non-factorizable decays of B mesons are much smaller than those for factorizable ones [20]. Therefore, it is easily understood that the measured rates for production of $D_{s0}^+(2317)$ in B_u^+ and B_d^0 decays are much lower than those for spectator (factorizable) $B \rightarrow D_s^+$ (or D_s^{*+}) \bar{D} decays [4]. Regarding with productions of \hat{F}_I^{++} and \hat{F}_I^0 (as well as \hat{F}_I^0), they are depicted by the same form of diagrams as seen in Fig. 1, so that it is qualitatively expected that their rates are not very far from each other, as discussed in our previous work [17]. However, this is in contrast to the recent result from the Belle experiment [16], as mentioned before. Therefore, the decay property of $D_{s0}^+(2317)$ and the productions of its partners seems to be inconsistent with each other.

To see more explicitly this, we study productions of $\hat{F}_I^{++} = D_{s0}^+(2317)$ and its partners, \hat{F}_I^0 and \hat{F}_I^{++} , in B_u^+ and B_d^0 decays, using a hard D meson approximation as an extension of a hard pion technique in the infinite momentum frame (IMF) [21] which can be considered as an innovation of the well-known soft-pion technique [22]. The effective weak Hamiltonian H_w which controls B decays producing charm-strange mesons is provided by $H_w = (G_F/\sqrt{2})V_{cs}V_{cb}\{c_1O_1 + c_2O_2\} + h.c.$, where O_1 and O_2 are given by $O_1 = :(\bar{c}s)_{V-A}(\bar{b}c)_{V-A}:$ and $O_2 = :(\bar{c}c)_{V-A}(\bar{b}s)_{V-A}:$, and c_1 and c_2 are their coefficients with QCD corrections. The CKM matrix elements V_{ij} 's [23] are taken to be real, because the CP invariance is always assumed in this note. It is known that measured rates for spectator (factorizable) decays of charm and B mesons are reproduced in a good approximation in the Bauer-Stech-Wirbel (BSW) scheme [20] in which the effective weak Hamiltonian H_w is redefined as, $H_w \rightarrow H_w^{\text{BSW}} + \tilde{H}_w$, where H_w^{BSW} is the so-called BSW Hamiltonian [24] and is given in the form $H_w^{\text{BSW}} = (G_F/\sqrt{2})V_{cs}V_{cb}\{a_1O_1 + a_2O_2\} + h.c.$. Here, the coefficients of O_1 and O_2 are given by $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$ with the color degree of freedom N_c . The terms proportional to $1/N_c$ are extracted from H_w by using the Fierz reordering, and the four-quark operators O_1 and O_2 should be no longer Fierz reordered. In the vacuum insertion prescription, matrix elements of H_w^{BSW} taken between the initial and final hadron states are factorizable. The extra term \tilde{H}_w after extracting a part of H_w^{BSW} (proportional to $1/N_c$) from H_w is written as $\tilde{H}_w = (G_F/\sqrt{2})V_{cs}V_{cb}\{c_2\tilde{O}_1 + c_1\tilde{O}_2\} + h.c.$, where $\tilde{O}_1 = 2\sum_a :(\bar{c}t^a s)_{V-A}(\bar{b}t^a c)_{V-A}:$ and $\tilde{O}_2 = 2\sum_a :(\bar{c}t^a c)_{V-A}(\bar{b}t^a s)_{V-A}:$ with the generator t^a of the color $SU_c(N_c)$. While \tilde{H}_w is taken away under the vacuum insertion prescription, it now survives and provides dynamical contributions of hadrons to hadronic weak decays. (For more details, see [25–27].)

Now, we study productions of $\hat{F}_I^{++} = D_{s0}^+(2317)$ and its partners in B decays. In the hard D meson approximation in the IMF, it is assumed that the (non-factorizable) amplitude for $B(p) \rightarrow \hat{F}(p')\bar{D}(q)$ is approximately given by

$$M(B \rightarrow \hat{F}\bar{D}) \simeq \lim_{p \rightarrow \infty, q \rightarrow 0} M(B \rightarrow \hat{F}\bar{D}) = M_{ETC}(B \rightarrow \hat{F}\bar{D}) + M_S(B \rightarrow \hat{F}\bar{D}) \quad (3)$$

under the partially conserved axial-vector current (PCAC) hypothesis, where B denotes B_u^+ or B_d^0 , and \hat{F} a charm-strange scalar tetra-quark meson, \hat{F}_I^0 , \hat{F}_I^{++} , \hat{F}_I^{++} or \hat{F}_0^{++} . In the above equation, M_{ETC} is the so-called equal-time

commutator term which is given by

$$M_{ETC}(B \rightarrow \hat{F}\bar{D}) = -\frac{i}{f_D} \langle \hat{F} | [V_D, \tilde{H}_w(0)] | B \rangle. \quad (4)$$

It has the same form as the ETC term in the soft pion approach but it now should be evaluated in the IMF. M_S is the surface term which is given in the form

$$M_S(B \rightarrow \hat{F}\bar{D}) = \lim_{p \rightarrow \infty, q \rightarrow 0} \left\{ -\frac{i}{f_D} q^\mu T_\mu \right\}, \quad (5)$$

where T_μ is the hypothetical amplitude

$$T_\mu = i \int e^{iqx} \langle \hat{F}(p') | T[A_\mu^{(D)}(x), \tilde{H}_w(0)] | B(p) \rangle d^4x, \quad (6)$$

and $A_\mu^{(D)}$ is the axial-vector current with the flavor of D . M_S disappears in the soft pseudoscalar meson (π, K, \dots) approximation but now survives [21], and is given by a sum of pole amplitudes,

$$M_S(B \rightarrow \hat{F}\bar{D}) = -\frac{i}{f_D} \left\{ \sum_n \left(\frac{m_{\hat{F}}^2 - m_B^2}{m_n^2 - m_B^2} \right) \langle \hat{F} | A_D | n \rangle \langle n | \tilde{H}_w | B \rangle - \sum_\ell \left(\frac{m_{\hat{F}}^2 - m_B^2}{m_\ell^2 - m_{\hat{F}}^2} \right) \langle \hat{F} | \tilde{H}_w | \ell \rangle \langle \ell | A_D | B \rangle \right\}, \quad (7)$$

where f_D denotes the decay constant of D . Here, V_D and A_D are the flavor charge and axial-charge with the flavor of D , respectively. In the above, we have used

$$\begin{cases} \left\{ \frac{\langle \hat{F}(\mathbf{p}') | V_\pi | n(\mathbf{p}_n) \rangle}{N_{\hat{F}} N_n} \right\}_{\mathbf{p}_n = \mathbf{p} \rightarrow \infty} &= (2\pi)^3 \delta^{(3)}(\mathbf{p}_n - \mathbf{p}') \langle \hat{F} | V_\pi | n \rangle, \\ \left\{ \frac{\langle \hat{F}(\mathbf{p}') | A_\pi | n(\mathbf{p}_n) \rangle}{N_{\hat{F}} N_n} \right\}_{\mathbf{p}_n = \mathbf{p} \rightarrow \infty} &= (2\pi)^3 \delta^{(3)}(\mathbf{p}_n - \mathbf{p}') \langle \hat{F} | A_\pi | n \rangle, \\ \left\{ \langle n(\mathbf{p}_n) | \tilde{H} | B(\mathbf{p}) \rangle \right\}_{\mathbf{p}_n = \mathbf{p} \rightarrow \infty} &= \langle n | \tilde{H} | B \rangle \end{cases} \quad (8)$$

with the normalization factors $N_{\hat{F}}$ and N_n of the state vectors. (For more details, see [21] and [25–27].) In the intermediate n and ℓ , single hadron states with the infinite momentum survive, and provide the s - and u -channel poles, respectively. Although M_S plays an important role in hadronic weak decays of K [26] and charm mesons [27], their contributions will be not very important in B decays, as long as M_{ETC} survives [28]. It is because the masses of B mesons are much higher than those of charm and light mesons, and therefore, mass dependent factors do not strongly enhance M_S in B decays [25]. In the present case, the $B \rightarrow \hat{F}\bar{D}$ decay under consideration is dominated by a hidden-charm strange tetra-quark state $\tilde{\kappa}^c(0^-) \sim \{cn\bar{c}\bar{s}\}$, ($n = u, d$) with $J^P = 0^-$ in the intermediate $|n\rangle$ and $\langle n|$, and a charmed bottom tetra-quark scalar state $\hat{B}_c \sim [nc][\bar{n}\bar{b}]$, ($n = u, d$) in the $|\ell\rangle$ and $\langle \ell|$. However, in the s -channel, the matrix element $\langle \hat{F} | A_D | \tilde{\kappa}^c(0^-) \rangle$ will be small, because the wavefunction overlap between the lowest tetra-quark state \hat{F} and the hypothetical $\tilde{\kappa}^c(0^-)$ state with some orbital excitation will be small. In the u -channel, the matrix element of A_D between the hypothetical $\langle \hat{B}_c |$ and the initial $|B\rangle$, i.e., the $\bar{B}\hat{B}_c\bar{D}$ coupling strength, will be small because of a variety of color and spin configurations in the tetra-quark state [6]. Therefore, contributions of M_S would not be important in decays under consideration, because M_{ETC} survives.

To evaluate amplitudes under consideration, we parametrize asymptotic matrix elements of V_D , using the asymptotic flavor $SU_f(4)$ symmetry (the $SU_f(4)$ symmetry of matrix elements of V_D taken between single hadron states with the infinite momentum) [21]. We here list their results which will be used later in this note,

$$\begin{aligned} \langle \hat{F}_I^{++} | V_{D^+} | \hat{\kappa}^{c+} \rangle &= \langle \hat{F}_I^0 | V_{D^0} | \hat{\kappa}^{c0} \rangle = \sqrt{2} \langle \hat{F}_I^+ | V_{D^0} | \hat{\kappa}^{c+} \rangle = \sqrt{2} \langle \hat{F}_0^+ | V_{D^0} | \hat{\kappa}^{c+} \rangle \\ &= -\sqrt{2} \langle \hat{F}_I^+ | V_{D^+} | \hat{\kappa}^{c0} \rangle = -\sqrt{2} \langle \hat{F}_0^+ | V_{D^+} | \hat{\kappa}^{c0} \rangle = \dots = \langle \hat{F}_I^{++} | V_{\pi^0} | \hat{F}_I^{++} \rangle = 1, \end{aligned} \quad (9)$$

where $\hat{\kappa}^c$'s denote hidden-charm strange members of scalar tetra-quark mesons in our model [5], i.e., $\hat{\kappa}^c \sim [cn][\bar{c}\bar{s}]$, ($n = u, d$). Here, it should be noted that the above parametrization might cause about 20–30 per cent errors in amplitudes. It is because matrix elements of V_D taken between single hadron states are given by a form factor of a charm changing vector current at zero momentum transfer squared which is normalized to be unity in the $SU_f(4)$ symmetry limit, while the measured form factors have been given as [29]

$$f_+^{(\bar{K}D)}(0) = 0.74 \pm 0.03 \quad \text{and} \quad \frac{f_+^{(\pi D)}(0)}{f_+^{(\bar{K}D)}(0)} = \begin{cases} 1.00 \pm 0.11 \pm 0.02 & (\text{Fermilab E687}), \\ 0.99 \pm 0.08 & (\text{CLEO}), \end{cases} \quad (10)$$

where the results from Fermilab E687 and CLEO have been given in [30] and [31], respectively. Regarding with a parametrization of asymptotic matrix elements of A_D , however, we skip to list their results, because we have neglected contributions of M_S which contains matrix elements of axial-charges. In this way, we here list amplitudes for productions of charm-strange scalar tetra-quark mesons in B decays as

$$M(B_u^+ \rightarrow \hat{F}_I^{++} D^-) \simeq -\frac{i}{f_D} \left\{ \langle \hat{\kappa}^{c+} | \tilde{H}_w | B_u^+ \rangle + \dots \right\}, \quad (11)$$

$$M(B_d^0 \rightarrow \hat{F}_I^0 \bar{D}^0) \simeq -\frac{i}{f_D} \left\{ \langle \hat{\kappa}^{c0} | \tilde{H}_w | B_d^0 \rangle + \dots \right\}, \quad (12)$$

$$M(B_u^+ \rightarrow \hat{F}_I^+ \bar{D}^0) \simeq -\sqrt{\frac{1}{2}} \frac{i}{f_D} \left\{ \langle \hat{\kappa}^{c+} | \tilde{H}_w | B_u^+ \rangle + \dots \right\}, \quad (13)$$

$$M(B_u^+ \rightarrow \hat{F}_0^+ \bar{D}^0) \simeq -\sqrt{\frac{1}{2}} \frac{i}{f_D} \left\{ \langle \hat{\kappa}^{c+} | \tilde{H}_w | B_u^+ \rangle + \dots \right\}, \quad (14)$$

$$M(B_d^0 \rightarrow \hat{F}_I^+ D^-) \simeq \sqrt{\frac{1}{2}} \frac{i}{f_D} \left\{ \langle \hat{\kappa}^{c0} | \tilde{H}_w | B_d^0 \rangle + \dots \right\}, \quad (15)$$

$$M(B_d^0 \rightarrow \hat{F}_0^+ D^-) \simeq \sqrt{\frac{1}{2}} \frac{i}{f_D} \left\{ \langle \hat{\kappa}^{c0} | \tilde{H}_w | B_d^0 \rangle + \dots \right\}, \quad (16)$$

in the approximation in which M_S is neglected, where the ellipses denote the neglected contributions of M_S . As seen in Eqs. (11) – (16), these amplitudes are given by $\langle \hat{\kappa}^{c+} | \tilde{H}_w | B_u^+ \rangle$ or $\langle \hat{\kappa}^{c0} | \tilde{H}_w | B_d^0 \rangle$. Difference between the above two matrix elements of \tilde{H}_w is in their spectator quarks, i.e., u in the former and d in the latter. Therefore, these two matrix elements are equivalent to each other in the $SU_I(2)$ symmetry limit. With these approximations, we obtain the following relations of rates for productions of charm-strange scalar tetra-quark mesons,

$$\begin{aligned} \Gamma(B_u^+ \rightarrow \hat{F}_I^{++} D^-) &\simeq 2\Gamma(B_u^+ \rightarrow \hat{F}_0^+ \bar{D}^0) \simeq 2\Gamma(B_u^+ \rightarrow \hat{F}_I^+ \bar{D}^0) \\ &\simeq \Gamma(B_d^0 \rightarrow \hat{F}_I^0 \bar{D}^0) \simeq 2\Gamma(B_d^0 \rightarrow \hat{F}_0^+ D^-) \simeq 2\Gamma(B_d^0 \rightarrow \hat{F}_I^+ D^-), \end{aligned} \quad (17)$$

when masses of \hat{F}_0^+ , \hat{F}_I^0 , \hat{F}_I^+ and \hat{F}_I^{++} are approximately degenerate.

Because the measured lifetimes of B_u^+ and B_d^0 are not equal to each other [4],

$$(\tau_{B_u^+})_{\text{exp}} = (1.641 \pm 0.008) \times 10^{-12} \text{ s} \quad \text{and} \quad (\tau_{B_d^0})_{\text{exp}} = (1.519 \pm 0.007) \times 10^{-12} \text{ s}, \quad (18)$$

however, we obtain separately

$$\mathcal{B}(B_u^+ \rightarrow \hat{F}_I^{++} D^-) \simeq 2\mathcal{B}(B_u^+ \rightarrow \hat{F}_0^+ \bar{D}^0) \simeq 2\mathcal{B}(B_u^+ \rightarrow \hat{F}_I^+ \bar{D}^0) = 2 \times (7.3_{-1.7}^{+2.2}) \times 10^{-4} \quad (19)$$

and

$$\mathcal{B}(B_d^0 \rightarrow \hat{F}_I^0 \bar{D}^0) \simeq 2\mathcal{B}(B_d^0 \rightarrow \hat{F}_0^+ D^-) \simeq 2\mathcal{B}(B_d^0 \rightarrow \hat{F}_I^+ D^-) = 2 \times (9.7_{-3.3}^{+4.0}) \times 10^{-4}, \quad (20)$$

where their measured branching fractions [4] have been taken in the above. Although the measured $\mathcal{B}(B_u^+ \rightarrow \hat{F}_I^+ \bar{D}^0)$ and $\mathcal{B}(B_d^0 \rightarrow \hat{F}_I^+ D^-)$ in Eqs. (19) and (20) still have large uncertainties, they are compatible with our approximate equality, $\Gamma(B_u^+ \rightarrow \hat{F}_I^+ \bar{D}^0) \simeq \Gamma(B_d^0 \rightarrow \hat{F}_I^+ D^-)$, in Eq. (17). This seems to mean that our approach is not so far from the reality. However, the same equation predicts that rates for \hat{F}_I^{++} and \hat{F}_I^0 productions are of the same order of magnitude as (larger by about a factor two than) those of \hat{F}_I^+ , and also that rates for productions of the iso-singlet partner \hat{F}_0^+ in B_u^+ and B_d^0 decays are approximately equal to those of \hat{F}_I^+ productions, as expected qualitatively in our previous work [17]. In addition, if a peak around the mass of $D_{s0}^+(2317)$ in the $D_s^{*+}\gamma$ mass spectrum is observed, it would be an indication of \hat{F}_0^+ , because it should decay dominantly into $D_s^{*+}\gamma$ as discussed before. Therefore, the fact that experiments did not observe any signal of the $D_{s0}^+(2317) \rightarrow D_s^{*+}\gamma$ decay as seen in Eq. (2) (i.e., $D_{s0}^+(2317)$ favors its assignment into an iso-triplet state) is not compatible with the fact that the Belle collaboration did not observe any signal of $\hat{F}_I^{++} = z^{++}$ and $\hat{F}_I^0 = z^0$ in B decays. Nevertheless, observation of $D_{s0}^0(2317)$ and $D_{s0}^{*+}(2317)$ in B decays are strongly desired, because the recently observed $X^\pm(5568)$ can be interpreted as the iso-triplet bottom partners of $D_{s0}^+(2317)$.

So far, we have considered that $D_{s0}^+(2317)$ is an iso-triplet tetra-quark scalar meson. However, there exists an argument [32] that the rate for the isospin non-conserving $D_{s0}^+(2317) \rightarrow D_s^+\pi^0$ decay can overcome that for the radiative $D_{s0}^+(2317) \rightarrow D_s^{*+}\gamma$ against the hierarchy of hadron interactions, if $D_{s0}^+(2317)$ is a $\{DK\}$ molecule (even

if it is an iso-singlet state), i.e., the intermediate DK loop contributions enhance extraordinarily the amplitude for the isospin non-conserving $D_{s0}^+(2317) \rightarrow D_s^+ \pi^0$ decay, and as the result, this model leads to a ratio of decay rates compatible with the measured one, Eq. (2). By the way, in order that this model works well, it seems to be implicitly required that the constituent D and K are sufficiently compact (or local). In the above analysis, however, a size parameter $\Lambda_{D_{s0}(2317)}$ which parametrizes the distribution of the constituent D and K in $D_{s0}^+(2317)$ has been introduced, and the cases with $\Lambda_{D_{s0}(2317)} = 1-2$ GeV (compact) and ∞ (the local limit) have been investigated. This implies that $\Lambda_{D_{s0}(2317)}^{-1}$, which can be very crudely considered as the size of the $\{DK\}$ molecule, is smaller than the measured size of the constituent K meson, as seen below. It is considered that the size of K meson is approximately given by the charge radius of K^\pm , which can be determined by measurements of the $eK^\pm \rightarrow eK^\pm$ scattering. A typical result on the mean square charge radius of K^\pm has been provided as [33] $\langle r^2 \rangle_{K^\pm} = (0.34 \pm 0.05) \text{ fm}^2$, so that the charge radius is $\sqrt{\langle r^2 \rangle_{K^\pm}} \simeq 0.58 \text{ fm}$. This result is considerably larger than the maximum value $(\Lambda_{D_{s0}(2317)}^{-1})_{\text{max}} = 1 \text{ GeV}^{-1}$ considered in [32]. This implies that the constituent K meson in the $\{DK\}$ molecule under consideration is not sufficiently compact. Therefore, the $\{DK\}$ molecular picture of $D_{s0}^+(2317)$ in [32] seems to be unrealistic.

In summary we have discussed that the decay property of $D_{s0}^+(2317)$ favors its assignment to an iso-triplet scalar state, because of the hierarchy of hadron interactions. In this case, it is expected that there exist its neutral and doubly charged partners, $D_{s0}^0(2317) = \hat{F}_I^0$ and $D_{s0}^{++}(2317) = \hat{F}_I^{++}$. As seen in the quark-line diagrams, amplitudes for their productions, $B \rightarrow \hat{F} \bar{D}$'s, are non-factorizable, and therefore, their rates have been calculated by using the hard D meson approximation in the IMF, where \hat{F} denotes \hat{F}_I^0 , \hat{F}_I^+ , \hat{F}_I^{++} or \hat{F}_0^+ . As the result, we have seen that the expected rates for productions of \hat{F}_I^{++} and \hat{F}_I^0 are approximately equal to each other, and that they are of the same order of magnitude as that for the productions of $\hat{F}_I^+ = D_{s0}^+(2317)$ in B_u^+ and B_d^0 decays, in contrast to the result from the Belle collaboration that no signal of \hat{F}_I^{++} and \hat{F}_I^0 was observed in B decays. In addition, we have provided a brief comment on an iso-singlet $\{DK\}$ molecular model of $D_{s0}^+(2317)$ which leads to a ratio of rates consistent with the measured one, Eq. (2). That is, the constituent K meson seems to be not sufficiently compact in this model, so that this model seems to be unnatural. In this way, it has been discussed that the decay property of $D_{s0}^+(2317)$ and the productions of its neutral and doubly charged partners are not compatible with each other. Because $X^\pm(5568)$ which can be interpreted as the iso-triplet bottom partners of $D_{s0}^+(2317)$ have been discovered, however, it is strongly desired that experiments will observe $D_{s0}^0(2317)$ and $D_{s0}^{++}(2317)$.

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